Math 1B Discussion Problems 26 Feb

1. Find a formula for the general term a_n for each of the following sequences.

   (a) $3, 2, 1, 0, -1, ...$

   (b) $1, -4, 9, -16, 25, -36, ...$

   (c) $\frac{1}{9}, \frac{2}{12}, \frac{4}{15}, \frac{8}{18}, \frac{16}{21}, ...$

2. Determine whether each of the following sequences converges or diverges. If the sequence converges find its limit.

   (a) $a_n = 2 + (0.1)^n$

   (b) $a_n = n \cos(n\pi)$

   (c) $a_n = \frac{\sin^2 n}{2^n}$

   (d) $a_n = \sqrt[n]{n}$

   (e) $a_n = \frac{n!}{n^n}$

   (f) $a_n = \frac{n!}{3^n}$

   (g) $a_1 = 1, a_{n+1} = 3 - a_n$

   (h) $a_1 = 1, a_{n+1} = \frac{1}{3-a_n}$ (Hint: Show that this sequence is decreasing and bounded below)

3. * The Fibonacci sequence is defined by $a_1 = a_2 = 1, a_{n+2} = a_{n+1} + a_n$. Let $b_n = \frac{a_{n+1}}{a_n}$.

$$a_n : 1, 1, 2, 3, 5, 8, 13, 21, ..., b_n : \frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, ...$$

   Show that b_n converges to the 'golden ratio' $\frac{1+\sqrt{5}}{2}$.
   (Hint: Show that the odd terms b_{2n+1} are increasing and bounded above, while the even terms b_{2n} are decreasing and bounded below)